DESY 99-049
April 1999

CP Violation and Prospects at B Factories and Hadron Colliders

A. Ali

Deutsches Elektronen-Synchrotron DESY, Hamburg
Notkestrae 85, D-22603 Hamburg, FRG

Invited Talk; To be published in the Proceedings of the 13th Topical Conference on Hadron Collider Physics, TIFR, Mumbai, India, January 14 - 20, 1999

]

CP VIOLATION AND PROSPECTS AT B FACTORIES AND HADRON COLLIDERS

AHMED ALI

Deutsches Elektronen Synchrotron DESY, Notkestrae 85, D-22603 Hamburg, FRG
E-mail: ali@x4u2.desy.de

We review the information on the CKM matrix elements, unitarity triangle and CP-violating phases α, β and γ in the standard model which will be measured in the forthcoming experiments at B factories, HERA-B and hadron colliders. We also discuss two-body non-leptonic decays $B \to h_1 h_2$, with h_i being light mesons, which are interesting from the point of view of CP violation and measurements of these phases. Partial rate CP asymmetries are presented in a number of decay modes using factorization for the matrix elements of the operators in the effective weak Hamiltonian. Estimates of the branching ratios in this framework are compared with existing data on $B \to K\pi, \eta' K, K^*\pi, \rho\pi$ decays from the CLEO collaboration.

1 Introduction

We shall review the following three topics in quark flavour physics:

- An update of the Cabibbo-Kobayashi-Maskawa CKM matrix [1].

Here, the results of a global fit of the CKM parameters yielding present profiles of the unitarity triangle and CP-violating phases α, β and γ and their correlations in the standard model (SM) are summarized [2].

- Estimates of the CP-violating partial rate asymmetries for charmless non-leptonic decays $B \to h_1 h_2$, where h_1 and h_2 are light mesons, based on next-to-leading-order perturbative QCD and the factorization approximation in calculating the matrix elements of the operators in the effective Hamiltonian approach [3].

Here, we first discuss a general classification of the CP-violating asymmetries in non-leptonic B decays and then give updated numerical estimates for a fairly large number of two-body decays involving penguin- and tree-transitions [3]. Most of the decays considered here have branching ratios which are estimated to be in excess of 10^{-6} (and some in excess of 10^{-5}) and many have measurable CP asymmetries. Hence, they are of interest for experiments at B factories and hadron machines.

- Comparison of the branching ratios for $B \to h_1 h_2$ decays measured by the CLEO collaboration [4,5] with the factorization-based theoretical estimates of the same [6,7,8].

The interest in these decays lies in that they provide first information on the QCD penguin-amplitudes and the CKM-suppressed non-leptonic $b \to u$ transitions. Hence, they will provide complementary information on the CKM matrix elements. It is argued that present data supports the factorization approach though it is not conclusive.

2 SM Fits of the CKM Parameters and the CP-Violating Phases α, β and γ

Within the standard model (SM), CP violation is due to the presence of a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix V. We shall use the parametrization of the CKM matrix due to Wolfenstein [9]:

$$V \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} , \tag{1}$$

which has four *a priori* unknown parameters A, λ, ρ and η, where λ is the Cabibbo angle and η represents the Kobayashi-Maskawa phase. The allowed region in ρ–η space can be elegantly displayed using the so-called unitarity triangle (UT). While one has six such relations, resulting from the unitarity of the CKM matrix, the one written below has received particular attention:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ . \tag{2}$$

Using the form of the CKM matrix in Eq. (1), this can be recast as

$$\frac{V_{ub}^*}{\lambda V_{cb}} + \frac{V_{td}}{\lambda V_{cb}} = 1 \ , \tag{3}$$

which is a triangle relation in the complex plane (i.e. ρ–η space). Thus, allowed values of ρ and η translate into allowed shapes of the unitarity triangle.

The interior CP-violating angles α, β and γ can be measured through CP asymmetries in B decays. Likewise, some of these these angles can also be measured through the decay rates. Additional constraints come from CP violation in the kaon system ($|\epsilon|$), as well as B_s^0-$\overline{B_s^0}$ mixing. In future, the decays $B \to (X_s, X_d)\gamma$, $B \to (X_s, X_d)\ell^+\ell^-$ and $K \to \pi\nu\bar{\nu}$ will further constrain the CKM matrix.

2.1 Input Data

The experimental and theoretical data which presently constrain the CKM parameters λ, A, ρ and η are summarized below.

- $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$:

We recall that $|V_{us}|$ has been extracted with good accuracy from $K \to \pi e\nu$ and hyperon decays [10] to be $|V_{us}| = \lambda = 0.2196 \pm 0.0023$. The determination of $|V_{cb}|$ is based on the combined analysis of the inclusive and exclusive B decays [10]: $|V_{cb}| = 0.0395 \pm 0.0017$, yielding $A = 0.819 \pm 0.035$. The knowledge of the CKM matrix element ratio $|V_{ub}/V_{cb}|$ is based on the analysis of the end-point lepton energy spectrum in semileptonic decays $B \to X_u\ell\nu_\ell$ and the measurement of the exclusive semileptonic decays $B \to (\pi, \rho)\ell\nu_\ell$. Present measurements in both the inclusive and exclusive modes are compatible with [11]: $\left|\frac{V_{ub}}{V_{cb}}\right| = 0.093 \pm 0.014$. This gives $\sqrt{\rho^2+\eta^2} = 0.423 \pm 0.064$.

- $|\epsilon|, \hat{B}_K$:

The experimental value of $|\epsilon|$ is [10]:

$$|\epsilon| = (2.280 \pm 0.013) \times 10^{-3} \ . \tag{4}$$

In the standard model, $|\epsilon|$ is essentially proportional to the imaginary part of the box diagram for K^0-$\overline{K^0}$ mixing and is given by [12]

$$\begin{aligned} |\epsilon| = \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \hat{B}_K \left(A^2 \lambda^6 \eta\right) \big(y_c \left\{\hat{\eta}_{ct} f_3(y_c, y_t) - \hat{\eta}_{cc}\right\} \\ + \ \hat{\eta}_{tt} y_t f_2(y_t) A^2 \lambda^4 (1-\rho)\big), \end{aligned} \tag{5}$$

where $y_i \equiv m_i^2/M_W^2$, and the functions f_2 and f_3 are the Inami-Lim function [13]. Here, the $\hat{\eta}_i$ are QCD correction factors, calculated at next-to-leading order: ($\hat{\eta}_{cc}$) [14], ($\hat{\eta}_{tt}$) [15] and ($\hat{\eta}_{ct}$) [16]. The theoretical uncertainty in the expression for $|\epsilon|$ is in the renormalization-scale independent parameter $\hat{B}_K$, which represents our ignorance of the hadronic matrix element $\langle K^0 | (\overline{d}\gamma^\mu (1-\gamma_5) s)^2 | \overline{K^0} \rangle$. Recent calculations of $\hat{B}_K$ using lattice QCD methods are summarized at the 1998 summer conferences by Draper [17] and Sharpe [18], yielding

$$\hat{B}_K = 0.94 \pm 0.15. \tag{6}$$

- $\Delta M_d, f_{B_d}^2 \hat{B}_{B_d}$:

The present world average for ΔM_d is [19]

$$\Delta M_d = 0.471 \pm 0.016 \ (ps)^{-1} \ . \tag{7}$$

The mass difference ΔM_d is calculated from the B_d^0-$\overline{B_d^0}$ box diagram, dominated by t-quark exchange:

$$\Delta M_d = \frac{G_F^2}{6\pi^2} M_W^2 M_B \left(f_{B_d}^2 \hat{B}_{B_d}\right) \hat{\eta}_B y_t f_2(y_t) |V_{td}^* V_{tb}|^2 \ , \tag{8}$$

where, using Eq. (1), $|V_{td}^* V_{tb}|^2 = A^2 \lambda^6 [(1-\rho)^2 + \eta^2]$. Here, $\hat{\eta}_B$ is the QCD correction, which has the value $\hat{\eta}_B = 0.55$, calculated in the $\overline{MS}$ scheme [15].

For the B system, the hadronic uncertainty is given by $f_{B_d}^2 \hat{B}_{B_d}$. Present estimates of this quantity using lattice QCD yield $f_{B_d} \sqrt{\hat{B}_{B_d}} = (190 \pm 23)$ MeV in the quenched approximation [17,18]. The effect of unquenching is not yet understood completely. Taking the MILC collaboration estimates of unquenching would increase the central value of $f_{B_d} \sqrt{\hat{B}_{B_d}}$ by 21 MeV [20]. In the fits discussed here [2], the following range has been used

$$f_{B_d} \sqrt{\hat{B}_{B_d}} = 215 \pm 40 \text{ MeV} \ . \tag{9}$$

- $\Delta M_s, f_{B_s}^2 \hat{B}_{B_s}$:

Table 1. Data used in the CKM fits.

Parameter	Value
λ	0.2196
$\|V_{cb}\|$	0.0395 ± 0.0017
$\|V_{ub}/V_{cb}\|$	0.093 ± 0.014
$\|\epsilon\|$	$(2.280 \pm 0.013) \times 10^{-3}$
ΔM_d	$(0.471 \pm 0.016)\ (ps)^{-1}$
ΔM_s	$> 12.4\ (ps)^{-1}$
$\overline{m_t}(m_t(pole))$	(165 ± 5) GeV
$\overline{m_c}(m_c(pole))$	1.25 ± 0.05 GeV
$\hat{\eta}_B$	0.55
$\hat{\eta}_{cc}$	1.38 ± 0.53
$\hat{\eta}_{ct}$	0.47 ± 0.04
$\hat{\eta}_{tt}$	0.57
$\hat{B}_K$	0.94 ± 0.15
$f_{B_d}\sqrt{\hat{B}_{B_d}}$	215 ± 40 MeV
ξ_s	1.14 ± 0.06

The B_s^0-$\overline{B_s^0}$ box diagram is again dominated by t-quark exchange, and the mass difference between the mass eigenstates ΔM_s is given by a formula analogous to that of Eq. (8):

$$\Delta M_s = \frac{G_F^2}{6\pi^2} M_W^2 M_{B_s} \left(f_{B_s}^2 \hat{B}_{B_s} \right) \hat{\eta}_{B_s} y_t f_2(y_t) |V_{ts}^* V_{tb}|^2 \ . \tag{10}$$

Using the fact that $|V_{cb}| = |V_{ts}|$ (Eq. (1)), it is clear that one of the sides of the unitarity triangle, $|V_{td}/\lambda V_{cb}|$, can be obtained from the ratio of ΔM_d and ΔM_s,

$$\frac{\Delta M_s}{\Delta M_d} = \frac{\hat{\eta}_{B_s} M_{B_s} \left(f_{B_s}^2 \hat{B}_{B_s} \right)}{\hat{\eta}_{B_d} M_{B_d} \left(f_{B_d}^2 \hat{B}_{B_d} \right)} \left| \frac{V_{ts}}{V_{td}} \right|^2 . \tag{11}$$

The only real uncertainty in this quantity is the ratio of hadronic matrix elements $f_{B_s}^2 \hat{B}_{B_s} / f_{B_d}^2 \hat{B}_{B_d}$. Present estimate of this quantity is [17,18]:

$$\xi_s = 1.14 \pm 0.06 \ . \tag{12}$$

The present lower bound on ΔM_s is: $\Delta M_s > 12.4\ (\mathrm{ps})^{-1}$ (at 95% C.L.) [11].

There are two other measurements which should be mentioned here. First, the KTEV collaboration [21] has recently reported a measurement of direct CP violation in the K sector through the ratio ϵ'/ϵ, with

$$\mathrm{Re}(\epsilon'/\epsilon) = (28.0 \pm 3.0(\mathrm{stat}) \pm 2.6(\mathrm{syst}) \pm 1.0(\mathrm{MC\ stat})) \times 10^{-4} \ , \tag{13}$$

in agreement with the earlier measurement by the CERN experiment NA31 [22], which reported a value of $(23 \pm 6.5) \times 10^{-4}$ for the same quantity. The present

world average is $\mathrm{Re}(\epsilon'/\epsilon) = (21.8 \pm 3.0) \times 10^{-4}$. This combined result excludes the superweak model [23] by more than 7σ.

A great deal of theoretical effort has gone into calculating this quantity at next-to-leading order accuracy in the SM [24,25,26]. The result of this calculation can be summarized in the following form due to Buras and Silvestrini [27]:

$$\mathrm{Re}(\epsilon'/\epsilon) = \mathrm{Im}\lambda_t \left[-1.35 + R_s \left(1.1|r_Z^{(8)}|B_6^{(1/2)} + (1.0 - 0.67|r_Z^{(8)}|)B_8^{(3/2)}\right)\right] \ . \quad (14)$$

Here $\lambda_t = V_{td}V_{ts}^* = A^2\lambda^4\eta$ and $r_Z^{(8)}$ represents the short-distance contribution, which at the NLO precision is estimated to lie in the range $6.5 \leq |r_Z^{(8)}| \leq 8.5$ [24,25]. The quantities $B_6^{(1/2)} = B_6^{(1/2)}(m_c)$ and $B_8^{(3/2)} = B_8^{(3/2)}(m_c)$ are the matrix elements of the $\Delta I = 1/2$ and $\Delta I = 3/2$ operators O_6 and O_8, respectively, calculated at the scale $\mu = m_c$. Lattice-QCD [28] and the $1/N_c$ expansion [29] yield:

$$0.8 \leq B_6^{(1/2)} \leq 1.3, \qquad 0.6 \leq B_8^{(3/2)} \leq 1.0 \ . \quad (15)$$

Finally, the quantity R_s in Eq. (14) is defined as:

$$R_s \equiv \left(\frac{150 \text{ MeV}}{m_s(m_c) + m_d(m_c)}\right)^2 \ , \quad (16)$$

essentially reflecting the s-quark mass dependence. The present uncertainty on the CKM matrix element is $\pm 23\%$, which is already substantial. However, the theoretical uncertainties related to the other quantities discussed above are considerably larger. For example, the ranges $\epsilon'/\epsilon = (5.3 \pm 3.8) \times 10^{-4}$ and $\epsilon'/\epsilon = (8.5 \pm 5.9) \times 10^{-4}$, assuming $m_s(m_c) = 150 \pm 20$ MeV and $m_s(m_c) = 125 \pm 20$ MeV, respectively, have been quoted as the best representation of the status of ϵ'/ϵ in the SM [30]. These estimates are somewhat on the lower side compared to the data but not inconsistent.

Thus, whereas ϵ'/ϵ represents a landmark measurement, establishing for the first time direct CP-violation in decay amplitudes, and hence removing the superweak model of Wolfenstein and its various incarnations from further consideration, its impact on CKM phenomenology, particularly in constraining the CKM parameters, is marginal. For this reason, the measurement of ϵ'/ϵ is not included in the CKM fits summarized here.

Second, the CDF collaboration has recently made a measurement of $\sin 2\beta$ [31,32]. In the Wolfenstein parametrization, $-\beta$ is the phase of the CKM matrix element V_{td}. From Eq. (1) one can readily find that

$$\sin(2\beta) = \frac{2\eta(1-\rho)}{(1-\rho)^2 + \eta^2} \ . \quad (17)$$

Thus, a measurement of $\sin 2\beta$ would put a strong contraint on the parameters ρ and η. However, the CDF measurement gives [31]

$$\sin 2\beta = 0.79^{+0.41}_{-0.44} \ , \quad (18)$$

or $\sin 2\beta > 0$ at 93% C.L. This constraint is quite weak – the indirect measurements already constrain $0.52 \leq \sin 2\beta \leq 0.94$ at the 95% C.L. in the SM [2]. (The CKM fits reported recently in the literature [11,33,34] yield similar ranges.) In light of this, this measurement is not included in the fits. The data used in the CKM fits are summarized in Table 1.

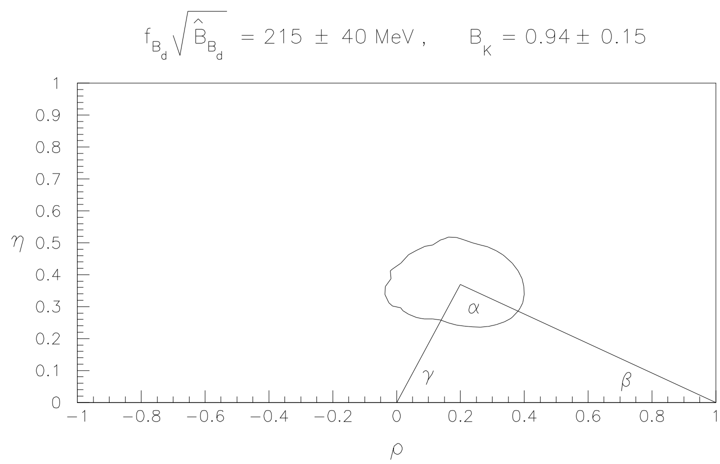

Figure 1. Allowed region in ρ–η space in the SM, from a fit to the ten parameters discussed in the text and given in Table 1. The limit on ΔM_s is included using the amplitude method [35]. The theoretical errors on $f_{B_d}\sqrt{\hat{B}_{B_d}}$, $\hat{B}_K$ and ξ_s are treated as Gaussian. The solid line represents the region with $\chi^2 = \chi^2_{min} + 6$ corresponding to the 95% C.L. region. The triangle shows the best fit. (From Ref. 2.)

2.2 SM Fits

In the fit presented here [2], ten parameters are allowed to vary: ρ, η, A, m_t, m_c, η_{cc}, η_{ct}, $f_{B_d}\sqrt{\hat{B}_{B_d}}$, $\hat{B}_K$, and ξ_s. The ΔM_s constraint is included using the amplitude method [35]. The rest of the parameters are fixed to their central values. The allowed (95% C.L.) ρ–η region is shown in Fig. 1. The best fit has $(\rho, \eta) = (0.20, 0.37)$.

The CP angles α, β and γ can be measured in CP-violating rate asymmetries in B decays. These angles can be expressed in terms of ρ and η. Thus, different shapes of the unitarity triangle are equivalent to different values of the CP angles. Referring to Fig. 1, we note that the preferred (central) values of these angles are $(\alpha, \beta, \gamma) = (93^\circ, 25^\circ, 62^\circ)$. The allowed ranges at 95% C.L. are

$$\begin{aligned} 65^\circ &\le \alpha \le 123^\circ \\ 16^\circ &\le \beta \le 35^\circ \\ 36^\circ &\le \gamma \le 97^\circ \end{aligned} \tag{19}$$

Of course, the values of α, β and γ are correlated, i.e. they are not all allowed simultaneously. We illustrate these correlations in Figs. 2 and 3. Fig. 2 shows the allowed region in $\sin 2\alpha$–$\sin 2\beta$ space allowed by the data. And Fig. 3 shows the allowed (correlated) values of the CP angles α and γ. This correlation is roughly linear, due to the relatively small allowed range of β (Eq. (19)).

We remark that the correlations shown are specific to the SM and are expected to be different, in general, in non-SM scenarios. A comparative study for some

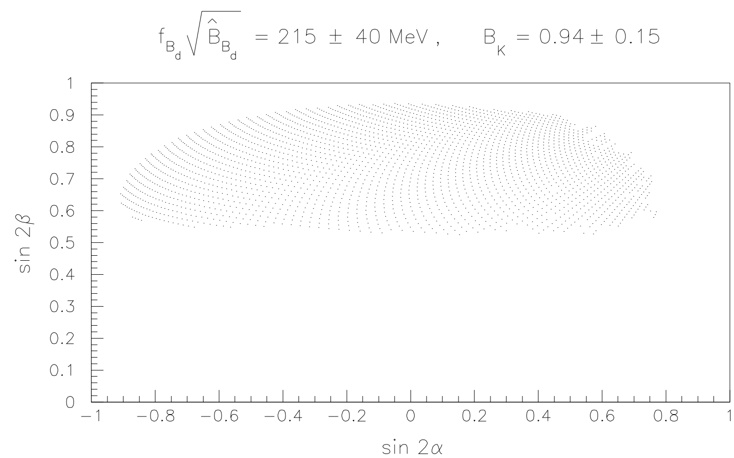

Figure 2. Allowed 95% C.L. region of the CP-violating quantities $\sin 2\alpha$ and $\sin 2\beta$ in the SM, from a fit to the data given in Table 1.

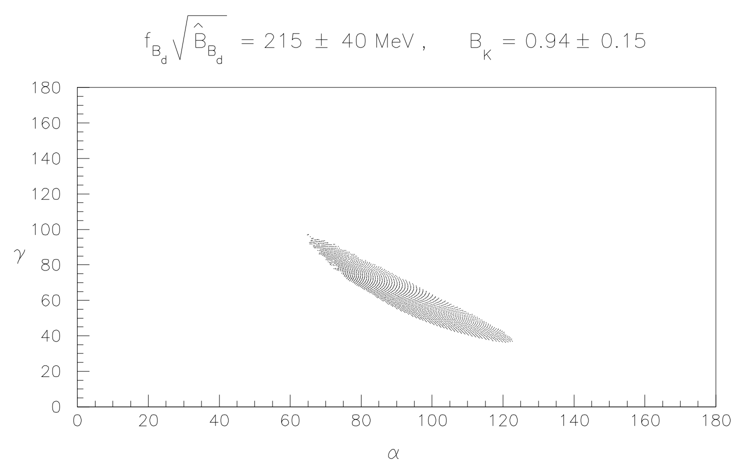

Figure 3. Allowed 95% C.L. region of the CP-violating quantities α and γ in the SM, from a fit to the data given in Table 1.

variants of the minimal supersymmetric model (MSSM) has been presented recently [2], underlying the importance of measuring the angles α, β and γ precisely. One expects almost similar constraints on β from the CKM fits in the SM and MSSM, but α and γ may provide a discrimination.

3 CP-Violating Asymmetries in $B^{\pm} \to (h_1 h_2)^{\pm}$ Decays

Apart from the decay modes $B \to J/\psi K_s^0$ and $B \to \pi\pi$, discussed at great length in the literature, there are many interesting two-body decays $B \to h_1 h_2$ which are expected to have large CP asymmetries in their partial decay rates. Recent measurements by the CLEO collaboration of B-decays into final state such as $h_1 h_2 = \pi K, \eta' K, \pi\rho, \pi K^*$ have rekindled theoretical interest in these decays [4,5]. A completely quantitative description of these and related two-body decays is a challenging enterprise, as this requires knowledge of the four-quark-operator matrix elements in the decays $B \to h_1 h_2$, for which the QCD technology is not yet ripe. Hence, calculations of the decay amplitudes from first principles in QCD are difficult and a certain amount of model-building is unavoidable. Here, we shall summarize the work done in estimating the rates [6,7,8] and CP asymmetries [3] in some selective decay modes, based on perturbative QCD and factorization.

For charged $B^{\pm}$ decays the CP-violating rate-asymmetries in partial decay rates are defined as follows:

$$A_{CP} \equiv \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)} \ , \tag{20}$$

where $f^{\pm} = (h_1 h_2)^{\pm}$. To be non-zero, these asymmetries require both weak and strong phase differences in interfering amplitudes. The weak phase difference arises from the superposition of amplitudes from the various tree- and penguin-diagrams, with the former involving $b \to u$ and the latter $b \to s$ or $b \to d$ transitions. The strong phases, which are needed to obtain non-zero values for A_{CP} in (20), are generated by final state interactions. This is modeled using perturbative QCD by taking into account the NLO corrections, following earlier suggestions along these lines [36]. It should be stressed that this formalism includes not only the so-called *charm penguins* [37] but *all* penguins (as well as the tree-contribution) in the framework of an effective Hamiltonian.

3.1 CP-violating Asymmetries Involving $b \to s$ Transitions

For the $b \to s$, and the charge conjugated $\bar{b} \to \bar{s}$, transitions, the respective decay amplitudes $\mathcal{M}$ and $\overline{\mathcal{M}}$, including the weak and strong phases, can be generically written as:

$$\begin{aligned} \mathcal{M} &= T\xi_u - P_{tc}\xi_t e^{i\delta_{tc}} - P_{uc}\xi_u e^{i\delta_{uc}}, \\ \overline{\mathcal{M}} &= T\xi_u^* - P_{tc}\xi_t^* e^{i\delta_{tc}} - P_{uc}\xi_u^* e^{i\delta_{uc}}, \end{aligned} \tag{21}$$

where we define

$$\begin{aligned} P_{tc}e^{i\delta_{tc}} &\equiv P_t e^{i\delta_t} - P_c e^{i\delta_c}, \\ P_{uc}e^{i\delta_{uc}} &\equiv P_u e^{i\delta_u} - P_c e^{i\delta_c}. \end{aligned} \tag{22}$$

Here $\xi_i = V_{ib}V_{is}^*$ and use has been made of the unitarity relation $\xi_c = -\xi_u - \xi_t$. In the above expressions T denotes the contributions from the current-current operators; P_t, P_c and P_u denote the contributions from penguin operators proportional

to the product of the CKM matrix elements ξ_t, ξ_c and ξ_u, and the corresponding strong phases are denoted by δ_t, δ_c and δ_u, respectively. The explicit expressions for the CP-violating asymmetry A_{CP} are, in general, not very illuminating [3]. However, as the amplitudes involve several small parameters[a], much simplified forms for A^- and A^+, and hence A_{CP}, can be obtained in specific decays by keeping only the leading terms [8,3].

To exemplify this, we note that $|\xi_u| \ll |\xi_t| \simeq |\xi_c|$, with an upper bound $|\xi_u|/|\xi_t| \leq 0.025$. In some channels, such as $B^\pm \to K^\pm\pi^0$, $K^{*\pm}\pi^0$, $K^{*\pm}\rho^0$, typical value of the ratio $|P_{tc}/T|$ is of $O(0.1)$, with both P_{uc} and P_{tc} comparable with typically $|P_{uc}/P_{tc}| = O(0.3)$. Using these approximations, the CP-violating asymmetry in $b \to s$ transitions can be expressed as

$$A_{CP} \simeq \frac{2z_{12}\sin\delta_{tc}\sin\gamma}{1+2z_2\cos\delta_{tc}\cos\gamma+z_2^2}, \tag{23}$$

where $z_2 = |\xi_u/\xi_t| \times T/P_{tc}$. Note that A_{CP} is approximately proportional to $\sin\gamma$, as pointed out by Fleischer and Mannel [38] in the context of the decay $B \to K\pi$. Due to the circumstance that the suppression due to $|\xi_u/\xi_t|$ is stronger than the enhancement due to T/P_{tc}, restricting the value of z_2, the CP-violating asymmetry for these kinds of decays are expected to be O(10%). Explicit calculations in model estimates confirm this pattern [3].

There are also decay modes with vanishing tree contributions, such as $B^\pm \to \pi^\pm K_S^0$, $\pi^\pm K^{*0}$, $\rho^\pm K^{*0}$. With $T=0$ and $|\xi_u| \ll |\xi_t|$, the CP-violating asymmetry can now be expressed as

$$A_{CP} \simeq 2\frac{P_{uc}}{P_{tc}}\left|\frac{\xi_u}{\xi_t}\right|\sin(\delta_{uc}-\delta_{tc})\sin\gamma. \tag{24}$$

As $P_{uc}/P_{tc} \ll 1$, and also $|\xi_u/\xi_t| \ll 1$, the CP-violating asymmetries are expected to be small. Some representative estimates are [3]: $A_{CP}(\pi^\pm K_s^0) = -1.5\%$, $A_{CP}(\pi^\pm \overset{(-)}{K}{}^{*0}) = -1.7\%$, $A_{CP}(\rho^\pm \overset{(-)}{K}{}^{*0}) = -1.7\%$. In scenarios with additional CP-violating phases, these CP asymmetries can be greatly enhanced and hence they are of interest in searching for non-SM CP-violation effects in B decays.

3.2 CP-violating Asymmetries Involving $b \to d$ Transitions

For $b \to d$ transitions, the decay amplitudes can be expressed as

$$\begin{aligned}\mathcal{M} &= T\zeta_u - P_{tc}\zeta_t e^{i\delta_{tc}} - P_{uc}\zeta_u e^{i\delta_{uc}},\\ \overline{\mathcal{M}} &= T\zeta_u^* - P_{tc}\zeta_t^* e^{i\delta_{tc}} - P_{uc}\zeta_u^* e^{i\delta_{uc}},\end{aligned} \tag{25}$$

where $\zeta_i = V_{ib}V_{id}^*$, and again the CKM unitarity has been used in the form $\zeta_c = -\zeta_t - \zeta_u$. For the tree-dominated decays involving $b \to d$ transitions, such as

[a] The smallness of these quantities reflects the CKM-suppression and/or QCD dynamics calculated in perturbation theory.

$B^\pm \to \pi^\pm \eta^{(\prime)}$, $\rho^\pm \eta^{(\prime)}$, $\rho^\pm \omega$, the relation $P_{uc} < P_{tc} \ll T$ holds, and the CP-violating asymmetry is approximately given by

$$A_{CP} \simeq \frac{-2 z_1 \sin\delta_{tc} \sin\alpha}{1 + 2 z_1 \cos\delta_{tc} \cos\alpha}, \tag{26}$$

with $z_1 = |\zeta_t/\zeta_u| \times T P_{tc}/T'^2$ and $T'^2 \equiv T^2 - 2 T P_{uc} \cos\delta_{uc}$. Note, the CP-violating asymmetry is approximately proportional to $\sin\alpha$ in this case. Concerning z_1, we note that the suppression due to $P_{tc} T / T'^2 \ll 1$ is accompanied with some enhancement from $|\zeta_t/\zeta_u|$ (the central value of this quantity is about 2.1 [2]), making the CP-violating asymmetry in this kind of decays to have a value $A_{CP} = $ (5-10)%.

For the decays with vanishing tree contribution, such as $B^\pm \to K^\pm K_S^0$, $K^\pm \overset{(-)}{K}{}^{*0}$, $K^{*\pm} \overset{(-)}{K}{}^{*0}$, the CP-violating asymmetry is approximately proportional to $\sin\alpha$ again,

$$A_{CP} = \frac{-2 z_3 \sin(\delta_{uc} - \delta_{tc}) \sin\alpha}{1 - 2 z_3 \cos(\delta_{uc} - \delta_{tc}) \cos\alpha + z_3^2}, \tag{27}$$

with $z_3 = |\zeta_u/\zeta_t| \times P_{uc}/P_{tc}$. As the suppression from $|\zeta_u/\zeta_t|$ and $|P_{uc}/P_{tc}|$ is not very strong, the CP-violating asymmetry are typically of order (10-20)%.

We list the estimated CP asymmetries and branching ratios (charge-conjugate averaged) in $B^\pm \to (h_1 h_2)^\pm$ decays in the upper half of Table 2, keeping only those decays which are expected to have branching ratios in excess of 10^{-6}. While the listed A_{CP} are not sensitive to the precise values of the form factors, the branching ratios are; the numbers given correspond to the BSW model [39]. We have indicated the uncertainty on A_{CP} resulting from the virtuality of the gluon $g(k^2) \to q_i \bar{q}_i$, influencing the absorptive parts of the amplitudes, for $k^2 = m_b^2/2 \pm 2$ GeV^2.

4 CP-violating Asymmetries in $B^0 \to (h_1 h_2)^0$ Decays

The CP asymmetries involving the neutral $B^0(\bar{B}^0)$ decays may require time-dependent measurements. Defining the time-dependent asymmetries as

$$A_{CP}(t) \equiv \frac{\Gamma(B^0(t) \to f) - \Gamma(\overline{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\overline{B}^0(t) \to \bar{f})}, \tag{28}$$

there are four cases that one encounters for neutral $B^0(\bar{B}^0)$ decays:

- case (i): $B^0 \to f$, $\bar{B}^0 \to \bar{f}$, where f or $\bar{f}$ is not a common final state of B^0 and $\bar{B}^0$, for example $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$.
- case (ii): $B^0 \to (f = \bar{f}) \leftarrow \bar{B}^0$ with $f^{CP} = \pm f$, involving final states which are CP eigenstates, i.e., decays such as $\bar{B}^0(B^0) \to \pi^+\pi^-, \pi^0\pi^0, K_S^0\pi^0$ etc.
- case (iii): $B^0 \to (f = \bar{f}) \leftarrow \bar{B}^0$, with f involving final states which are not CP eigenstates. They include decays into two vector mesons $B^0 \to (VV)^0$, as the VV states are not CP-eigenstates.

Table 2. CP-rate asymmetries A_{CP} and charge-conjugate-averaged branching ratios for some selected $B \to h_1 h_2$ decays, estimated in the factorization approach [3], updated for the central values of the CKM fits [2] $\rho = 0.20$, $\eta = 0.37$ and the factorization model parameters $\xi = 0.5$ and $k^2 = m_b^2/2 \pm 2$ GeV^2.

Decay Modes	CP-class	$A_{CP}(\%)$	$BR(\times 10^{-6})$
$B^\pm \to K^\pm \pi^0$	(i)	$-7.7^{-2.2}_{+4.0}$	10.0
$B^\pm \to K^{*\pm} \pi^0$	(i)	$-14.4^{-4.4}_{+8.2}$	4.3
$B^\pm \to K^{*\pm} \rho^0$	(i)	$-13.5^{-4.0}_{+7.5}$	4.8
$B^\pm \to \eta \pi^\pm$	(i)	$9.3^{+1.9}_{-4.1}$	5.5
$B^\pm \to \eta' \pi^\pm$	(i)	$9.4^{+2.1}_{-4.5}$	3.7
$B^\pm \to \eta \rho^\pm$	(i)	$3.1^{+0.7}_{-1.7}$	8.6
$B^\pm \to \eta' \rho^\pm$	(i)	$3.1^{+0.7}_{-1.8}$	6.2
$B^\pm \to \rho^\pm \omega$	(i)	$7.0^{+1.5}_{-3.4}$	21.0
$B^\pm \to \eta' K^\pm$	(i)	$-4.9^{-1.2}_{+2.1}$	23.0
$B^\pm \to \pi^\pm \rho^0$	(i)	$-3.1^{-0.9}_{+2.0}$	9.0
$B^\pm \to \eta K^\pm$	(i)	$7.0^{+2.9}_{-5.2}$	2.6
$B^\pm \to \eta K^{*\pm}$	(i)	$-10.5^{-3.1}_{+5.9}$	2.1
$B^\pm \to K^\pm \omega$	(i)	$-14.4^{-4.5}_{+8.1}$	3.2
$B^\pm \to \pi^\pm \omega$	(i)	$7.7^{+1.7}_{-3.7}$	9.5
$B^\pm \to K^{*\pm} \omega$	(i)	$-10.3^{-3.1}_{+5.6}$	11.0
$\overset{(-)}{B^0} \to K^{*\pm} \rho^\mp$	(i)	$-17.2^{-5.5}_{+9.8}$	5.4
$\overset{(-)}{B^0} \to K^\pm \pi^\mp$	(i)	$-8.2^{-2.3}_{+4.3}$	14.0
$\overset{(-)}{B^0} \to \pi^\pm K^{*\mp}$	(i)	$-17.2^{-5.5}_{+9.8}$	6.0
$\overset{(-)}{B^0} \to \eta' K^0_S$	(ii)	$33.6^{-0.2}_{+0.3}$	23.0
$\overset{(-)}{B^0} \to \pi^+ \pi^-$	(ii)	$25.4^{+0.2}_{-1.0}$	13.0
$\overset{(-)}{B^0} \to \pi^0 \pi^0$	(ii)	$-45.1^{-1.8}_{+6.4}$	0.4
$\overset{(-)}{B^0} \to K^0_S \pi^0$	(ii)	$39.6^{+0.5}_{-0.9}$	3.0
$\overset{(-)}{B^0} \to K^0_S \eta$	(ii)	$41.2^{+0.7}_{-1.1}$	1.0
$\overset{(-)}{B^0} \to K^0_S \phi$	(ii)	35.2	9.0
$\overset{(-)}{B^0} \to \rho^+ \rho^-$	(iii)	$17.1^{+0.1}_{-0.6}$	24.0
$\overset{(-)}{B^0} \to \rho^0 \rho^0$	(iii)	$-46.0^{-1.4}_{+4.3}$	1.0
$\overset{(-)}{B^0} \to \omega \omega$	(iii)	$56.5^{+1.3}_{-2.8}$	1.1
$B^0/\bar{B}^0 \to \rho^- \pi^+ / \rho^+ \pi^-$	(iv)	$13.9^{-0.5}_{+1.0}$	7.8
$B^0/\bar{B}^0 \to \rho^+ \pi^- / \rho^- \pi^+$	(iv)	$9.7^{-0.6}_{+0.9}$	29.0

- case (iv): $B^0 \to (f \& \bar{f}) \leftarrow \bar{B}^0$ with $f^{CP} \neq f$, i.e., both f and $\bar{f}$ are common final states of B^0 and $\bar{B}^0$, but they are not CP eigenstates. Decays $B^0/\bar{B}^0 \to \rho^+\pi^-$, $\rho^-\pi^+$ and $B^0/\bar{B}^0 \to K^{*0} K^0_S$, $\bar{K}^{*0} K^0_S$ are two interesting examples in this class.

Here case (i) is very similar to the charged $B^\pm$ decays, discussed above. For case (ii), and (iii), $A_{CP}(t)$ would involve B^0 - $\overline{B^0}$ mixing. Assuming $|\Delta\Gamma| \ll |\Delta m|$ and $|\Delta\Gamma/\Gamma| \ll 1$, which hold in the standard model for the mass and width differences Δm and $\Delta\Gamma$ in the neutral B-sector, one can express $A_{CP}(t)$ in a simplified form:

$$A_{CP}(t) \simeq a_{\epsilon'} \cos(\Delta m t) + a_{\epsilon+\epsilon'} \sin(\Delta m t) \ . \tag{29}$$

The quantities $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ depend on the hadronic matrix elements:

$$a_{\epsilon'} = \frac{1 - |\lambda_{CP}|^2}{1 + |\lambda_{CP}|^2}, \quad a_{\epsilon+\epsilon'} = \frac{-2Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \tag{30}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{\langle f | H_{eff} | \bar{B}^0 \rangle}{\langle f | H_{eff} | B^0 \rangle}. \tag{31}$$

For case (i) decays, the coefficient $a_{\epsilon'}$ determines $A_{CP}(t)$, and since no mixing is involved for these decays, the CP-violating asymmetry is independent of time. We shall call these, together with the CP asymmetries in charged $B^\pm$ decays, CP-class (i) decays. For case (ii) and (iii), one has to separate the $\sin(\Delta m t)$ and $\cos(\Delta m t)$ terms to get the CP-violating asymmetry $A_{CP}(t)$. The time-integrated asymmetries are:

$$A_{CP} = \frac{1}{1+x^2} a_{\epsilon'} + \frac{x}{1+x^2} a_{\epsilon+\epsilon'}, \tag{32}$$

with $x = \Delta m/\Gamma \simeq 0.73$ for the B^0 - $\overline{B^0}$ case.

Case (iv) also involves mixing but here one has to study the four time-dependent decay widths for $B^0(t) \to f$, $\bar{B}^0(t) \to \bar{f}$, $B^0(t) \to \bar{f}$ and $\bar{B}^0(t) \to f$ [40]. These time-dependent widths can be expressed by four basic matrix elements

$$\begin{aligned} g &= \langle f | H_{eff} | B^0 \rangle, \ h = \langle f | H_{eff} | \bar{B}^0 \rangle, \\ \bar{g} &= \langle \bar{f} | H_{eff} | \bar{B}^0 \rangle, \ \bar{h} = \langle \bar{f} | H_{eff} | B^0 \rangle, \end{aligned} \tag{33}$$

which determine the decay matrix elements of $B^0 \to f \& \bar{f}$ and of $\bar{B}^0 \to \bar{f} \& f$ at $t = 0$. By measuring the time-dependent spectrum of the decay rates of B^0 and $\bar{B}^0$, one can find the coefficients of the two functions $\cos \Delta m t$ and $\sin \Delta m t$ and extract the quantities $a_{\epsilon'}$, $a_{\epsilon+\epsilon'}$, $|g|^2 + |h|^2$, $a_{\bar{\epsilon}'}$, $a_{\epsilon+\bar{\epsilon}'}$ and $|\bar{g}|^2 + |\bar{h}|^2$ as well as Δm and Γ.

Estimates of $A_{CP}(\overset{(-)}{B^0} \to h_1 h_2)$, representing the decays belonging to the CP-classes (i) to (iv), together with the branching ratios averaged over the charge-conjugated modes, are given in Table 2. They have estimated branching ratios in excess of 10^{-6} (except for the decay $\overset{(-)}{B^0} \to \pi^0 \pi^0$). They also include the decay modes $\overset{(-)}{B^0} \to K^\pm \pi^\mp$, $\overset{(-)}{B^0} \to \eta' K_S^0$, $\overset{(-)}{B^0} \to \pi^\pm K^{*\mp}$, $B^0/\bar{B}^0 \to \rho^- \pi^+ / \rho^+ \pi^-$, $B^0/\bar{B}^0 \to \rho^+ \pi^- / \rho^- \pi^+$, whose branching ratios have been measured by the CLEO collaboration. The CP asymmetries in all these partial decay rates are expected to be large.

Table 3. Branching ratios measured by the CLEO collaboration [4] and factorization-based theoretical estimates of the same [8] (in units of 10^{-5}), updated for the central values of the CKM fits [2] $\rho = 0.20$, $\eta = 0.37$ and the factorization model parameter $\xi = 0.5$. Theory numbers correspond to using the BSW model [39] [Lattice QCD/QCD sum rule] for the form factors.

Decay Mode	BR (Expt)[4]	BR(Theory)[8]
$B^0 \to K^+\pi^-$	$1.4 \pm 0.3 \pm 0.2$	1.4[1.7]
$B^+ \to K^+\pi^0$	$1.5 \pm 0.4 \pm 0.3$	1.0[1.1]
$B^+ \to K^0\pi^+$	$1.4 \pm 0.5 \pm 0.2$	1.6[1.9]
$B^+ \to \eta' K^+$	$7.4^{+0.8}_{-1.3} \pm 1.0$	2.3[2.7]
$B^0 \to \eta' K^0$	$5.9^{+1.8}_{-1.6} \pm 0.9$	2.3[2.7]
$B^0 \to \pi^- K^{*+}$	$2.2^{+0.8}_{-0.6}\,^{+0.4}_{-0.5}$	0.6[0.7]
$B^+ \to \pi^+\rho^0$	$1.5 \pm 0.5 \pm 0.4$	0.9[1.0]
$B^0/\bar{B}^0 \to \rho^-\pi^+$	$3.5^{+1.1}_{-1.0} \pm 0.5$	3.7[4.3]

5 Comparison of the Factorization Model with the CLEO Data

Before we discuss the numerical results, a technical remark on the underlying theoretical framework is in order. The estimates being discussed here [8,3], and the earlier work along these lines [6,7], are all based on using the NLO virtual corrections for the matrix elements of the four-quark operators, calculated in the Landau gauge with off-shell quarks [41]. This renders the effective (phenomenological) coefficients used in these works both gauge-and quark (off-shell) mass-dependent [42]. The remedy for this unsatisfactory situation is to replace the virtual corrections with the ones calculated with on-shell quarks, which are manifestly gauge invariant. A proof that gauge- and renormalization-scheme-independent effective coefficients (and hence decay amplitudes) can be consistently obtained in perturbation theory now exists [43]. Further discussion of these aspects and alternative derivation of the gauge-invariant on-shell amplitudes in $B \to h_1 h_2$ decays will be reported elsewhere [46]. From the phenomenological point of view, the corrections in the effective coefficients are, however, small [44,45]. So, the real big unknown in this approach is the influence of the neglected soft non-factorizing contributions.

We now compare the predictions of the factorization-based estimates with the CLEO data. Since no CP asymmetries in the decays $B \to h_1 h_2$ have so far been measured, this comparison can only be done in terms of the branching ratios. We give in Table 3 eight $B \to h_1 h_2$ decay modes, their branching ratios measured by CLEO [4] and the updated theoretical estimates of the same [8]. The entries given for $B^0/\bar{B}^0 \to \rho^-\pi^+$ are for the sum of the two modes, as CLEO does not distinguish between the decay products of B^0 and $\bar{B}^0$. Moreover, we have fixed the factorization model parameter to $\xi = 0.5$, which is suggested by the measured $B \to \pi\rho$ branching ratios. Table 3 shows that all five $K\pi$ and $\pi\rho$ decay modes are well-accounted for in the factorization-model, but the $\pi^\pm K^{*\mp}$ and the two $B \to \eta' K$ modes are typically a factor 2 below the CLEO data, though experimental errors are large to be conclusive. Also, there is some dependence of the decay rates on the form factors, as indicated by the two set of numbers corresponding to the use

of the BSW-model [39] and the Lattice-QCD/QCD-sum-rule-based estimates [8], and on ξ.

While the final verdict of the experimental jury is not yet in, it is fair to say that the factorization approach, embedded in a well-defined perturbative framework, provides a description of the present data which is accurate within a factor 2. This is a hint that non-factorizing soft final state interactions do not represent a dominant theme in B decays. Present data gives some credibility to the idea that perturbative QCD-based methods can be used in estimating final state interactions. Non-leptonic $B \to h_1 h_2$ decays discussed here, many of which will be measured in experiments at the B factories and hadron machines, will test this quantitatively.

Acknowledgements:

I thank Gustav Kramer, David London and Cai-Dian Lü for very productive collaborations, many helpful discussions and input to this report. Thanks are also due to Frank Würthwein for the communication and discussion of the CLEO data. The warm hospitality of the organizers of the Hadron13 conference is thankfully acknowledged.

References

1. N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and K. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
2. A. Ali and D. London, preprint DESY 99-042, UdeM-GPP-TH-99-58, hep-ph/9903535 (to be published in the European Physical Journal C).
3. A. Ali, G. Kramer and C.-D. Lü, *Phys. Rev.* D **59**, 014005 (1998).
4. Y. Gao and F. Würthwein (CLEO Collaboration), preprint CALT 68-2220, HUTP -99/A201, hep-ex/9904008 (1999).
5. For an excellent review on penguins in B meson decays and references to experimental and theoretical work, see K. Lingel, T. Skwarnicki and J.G. Smith, preprint COLO-HEP-395, SLAC-PUB-7796, HEPSY 98-1, hep-ex/9804015 (submitted to Ann. Rev. of Nucl. and Part. Science).
6. A. Ali and C. Greub, *Phys. Rev.* D **57**, 2996 (1998).
7. A. Ali, J. Chay, C. Greub and P. Ko, *Phys. Lett.* B **424**, 161 (1998).
8. A. Ali, G. Kramer and C.-D. Lü, *Phys. Rev.* D **58**, 094009 (1998).
9. L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983).
10. C. Caso et al. (Particle Data Group), *Eur. Phys. J.* C **3**, 1 (1998).
11. F. Parodi, in *Proc. of XXIXth. Int. Conf. on High Energy Physics*, Vancouver, B.C., (1998).
12. A.J. Buras, W. Slominski, and H. Steger, *Nucl. Phys.* B **238**, 529 (1984); *ibid.* B **245**, 369 (1984).
13. T. Inami and C.S. Lim, *Progr. Theor. Phys.* **65**, 297 (1981).
14. S. Herrlich and U. Nierste, *Nucl. Phys.* B **419**, 292 (1994).
15. A.J. Buras, M. Jamin and P.H. Weisz, *Nucl. Phys.* B **347**, 491 (1990).
16. S. Herrlich and U. Nierste, *Phys. Rev.* D **52**, 6505 (1995).
17. T. Draper, preprint hep-lat/9810065 (1998).

18. S. Sharpe, preprint hep-lat/9811006 (1998).
19. J. Alexander, *in Proc. of XXIXth. Int. Conf. on High Energy Physics*, Vancouver, B.C., (1998).
20. C. Bernard et al., *Phys. Rev. Lett.* **81**, 4812 (1998).
21. Seminar presented by P. Shawhan (KTEV Collaboration), Fermilab, IL, Feb. 24, 1999; see also http://fnphyx-www.fnal.gov/experiments/ktev/epsprime/.
22. G.D. Barr *et al.* (NA31 Collaboration), *Phys. Lett.* B **317**, 233 (1993).
23. L. Wolfenstein, *Phys. Rev. Lett.* **13**, 380 (1964).
24. A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* B **370**, 69 (1992); *ibid.* B **400**, 37 (1993); *ibid.* B **400**, 75 (1993); *ibid.* B **408**, 209 (1993).
25. M. Ciuchini *et al.*, *Phys. Lett.* B **301**, 263 (1993); *Nucl. Phys.* B **415**, 403 (1994).
26. S. Bertolini, M. Fabbrichesi and J.O. Egg, *Nucl. Phys.* B **499**, 197 (1995); *ibid.* B **476**, 225 (1996).
27. A.J. Buras and L. Silvestrini, preprint TUM-HEP-334/98, hep-ph/9811471 (1998).
28. For recent reviews, see R. Gupta, preprint hep-ph/9801412 and L. Conti *et al.*, *Phys. Lett.* B **421**, 273 (1998).
29. W.A. Bardeen, A.J. Buras and J.M. Gerard, *Phys. Lett.* B **180**, 133 (1986); *Nucl. Phys.* B **293**, 787 (1987); *Phys. Lett.* B **192**, 138 (1987); G.O. Köhler *et al., Phys. Rev.* D **58**, 014017 (1998).
30. A.J. Buras *et al.*, preprint TUM-HEP-316-98, hep-ph/9806471 (1998).
31. CDF Collaboration, report CDF/PUB/BOTTOM/CDF/4855 (1999).
32. G. Bauer, these proceedings.
33. S. Mele, preprint CERN-EP/98-133, hep-ph/9810333 (1998).
34. F. Parodi, P. Roudeau and A. Stocchi, preprint LAL 99-03, DELPHI 99-27 CONF 226, hep-ex/9903063 (1999).
35. H.G. Moser and A. Roussarie, *Nucl. Inst. Math.* A **384**, 491 (1997).
36. M. Bander, D. Silverman and A. Soni, *Phys. Rev. Lett.* **43**, 242 (1979); J. M. Gerard and W. S. Hou, *Phys. Rev.* D **43**, 2909 (1991); H. Simma, G. Eilam and D. Wyler, *Nucl. Phys.* B **352**, 367 (1991).
37. M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, *Nucl. Phys.* B **501**, 271 (1997); M. Ciuchini *et al.*, *ibid.* B **512**, 3 (1998).
38. R. Fleischer and T. Mannel, *Phys. Rev.* D **57**, 2752 (1998).
39. M. Bauer, B. Stech and M. Wirbel, *Z. Phys.* C **34**, 103 (1987).
40. M. Gronau, *Phys. Lett.* B **233**, 479 (1989); R. Aleksan, I. Dunietz, B. Kayser and F. Le Diberder, *Nucl. Phys.* B **361**, 141 (1991); W.F. Palmer and Y.L. Wu, *Phys. Lett.* B **350**, 245 (1995).
41. A.J. Buras *et al.*, *Nucl. Phys.* B **370**, 69 (1992); M. Ciuchini *et al.*, *ibid.* B **415**, 403 (1994).
42. A.J. Buras and L. Silvestrini, preprint TUM-HEP-315-98, hep-ph/9806278.
43. H.Y. Cheng, H.n. Li and K.C. Yang, preprint hep-ph/9902239 (1999).
44. Y.-H. Chen, H.-Y. Cheng, B. Tseng and K.-C. Yang, preprint IP-ASTP-01-99, hep-ph/9903453 (1999).
45. A. Ali, G. Kramer and C.-D. Lü (in preparation).
46. A. Ali, J. Chay, C. Greub and P. Ko (in preparation).